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THE REVIVAL STRUCTURE OF RYDBERG WAVE PACKETS BEYOND THE REVIVAL TIME¹

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After a Rydberg wave packet forms, it is known to undergo a series of collapses and revivals within a time period called the revival time t_{rev} , at the end of which it resembles its original shape. We study the behavior of Rydberg wave packets on time scales much greater than t_{rev} . We find that after a few revival cycles the wave packet ceases to reform at multiples of the revival time. Instead, a new series of collapses and revivals commences, culminating after a time period $t_{\text{sr}} \gg t_{\text{rev}}$ with the formation of a wave packet that more closely resembles the initial packet than does the full revival at time t_{rev} . Furthermore, at times that are rational fractions of t_{sr} , we show that the motion of the wave packet is periodic with periodicities that can be expressed as fractions of the revival time t_{rev} . These periodicities indicate a new type of fractional revival, occurring for times much greater than t_{rev} . We also examine the effects of quantum defects and laser detunings on the revival structure of Rydberg wave packets for alkali-metal atoms.

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When a Rydberg atom is excited by a short laser pulse, a state is created that has classical behavior for a limited time [1]. The wave packet initially oscillates with the classical keplerian period $T_{\text{cl}} = 2\pi\bar{n}^3$, where \bar{n} is the mean value of the principal quantum number excited in the packet. However, the motion is not entirely classical because the wave packet disperses with time. After many Kepler orbits, at the revival time t_{rev} the wave packet recombines nearly into its original shape. Moreover, prior to this full revival, the wave function evolves through a sequence of fractional revivals. Experiments have detected all stages of the evolution of the wave packet during the time t_{rev} , including the initial classical motion, the full revival, and the fractional revivals [2].

In this paper, we summarize results concerning the time evolution and revival structure of Rydberg wave packets on time scales much greater than the revival time t_{rev} . We have found that a new system of full and fractional revivals occurs for times beyond the revival time, with structure different from that of the usual fractional revivals. We have also examined the effects of quantum defects and laser detunings on the revival structure of Rydberg wave packets for alkali-metal atoms. The results summarized here have been published in refs. [3,4].

The time-dependent wave function for a Rydberg wave packet may be written as an expansion in terms of hydrogenic energy eigenstates:

$$\Psi(\vec{r}, t) = \sum_n c_n \varphi_n(\vec{r}) \exp(-iE_n t) \quad , \quad (1)$$

where $\varphi_n(\vec{r})$ is a hydrogenic wave function and $c_n = \langle \Psi(0) | \varphi_n \rangle$ is a weighting coefficient. We expand E_n around \bar{n} :

$$E_n \simeq E_{\bar{n}} + E'_{\bar{n}}(n - \bar{n}) + \frac{1}{2}E''_{\bar{n}}(n - \bar{n})^2 + \frac{1}{6}E'''_{\bar{n}}(n - \bar{n})^3 + \dots \quad , \quad (2)$$

where each prime on $E_{\bar{n}}$ denotes a derivative. This expansion defines three distinct time scales: $T_{\text{cl}} = \frac{2\pi}{E'_{\bar{n}}} = 2\pi\bar{n}^3$, $t_{\text{rev}} = \frac{-2\pi}{\frac{1}{2}E''_{\bar{n}}} = \frac{2\bar{n}}{3}T_{\text{cl}}$, and $t_{\text{sr}} = \frac{2\pi}{\frac{1}{6}E'''_{\bar{n}}} = \frac{3\bar{n}}{4}t_{\text{rev}}$. These time scales determine the evolution and revival structure of the wave packet. The first

¹Paper presented by R.B.

two are the usual time scales relevant in the description of the conventional revival structure. We include the third-order term because we are interested in times much greater than the revival time. This term defines the new time scale, which we refer to as the superrevival time t_{sr} .

By examining the effects of all three terms in the expansion of the wave function, we have found that at certain times t_{frac} it is possible to expand the wave function $\Psi(\vec{r}, t)$ as a series of subsidiary wave functions. We find that when $t_{\text{frac}} \approx \frac{1}{q}t_{\text{sr}}$, where q is an integer multiple of 3, the wave packet can be written as a sum of macroscopically distinct wave packets. Furthermore, at these times t_{frac} we also find that the motion of the wave packet is periodic with a period $T_{\text{frac}} \approx \frac{3}{q}t_{\text{rev}}$. Note that these periodicities are different from those of the fractional revivals, and thus a new level of revivals commences for $t > t_{\text{rev}}$. We also find that at the particular time $t_{\text{frac}} \approx \frac{1}{6}t_{\text{sr}}$, a single wave packet forms that resembles the initial wave packet more closely than the full revival does at time t_{rev} , i.e., a superrevival occurs.

These results can be generalized to include the effects of quantum defects, corresponding to nonhydrogenic energies E_{n^*} , and also to the case where the laser excites a mean energy corresponding to a noninteger value N^* . We find that the effects of quantum defects on the occurrence times and periodicities of long-term revivals are different from those of the laser detunings. A laser detuning cannot be mimicked by quantum defects or vice versa. Furthermore, the modification to the long-term revival times induced by the quantum defects cannot be obtained by direct rescaling of the hydrogenic results.

It is feasible that an experiment can be performed to detect the full and fractional revival structure for $t \gg t_{\text{rev}}$ discussed in this paper. One possibility is to use the pump-probe time-delayed photoionization method of detection for radial Rydberg wave packets excited in alkali-metal atoms with $\bar{n} \approx 45 - 50$, provided a delay line of 3 – 4 nsec is installed in the apparatus. For smaller values of \bar{n} , the required delay times can be reduced below 1 nsec. With $\bar{n} \simeq 36$, for example, the full/fractional superrevivals could be detected with delay lines used currently in experiments.

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